Fundamentals of Convection

→ Heat transfer through a solid is always by conduction, since the molecules of a solid remains at relatively fixed position

→ Heat transfer through a liquid & gas, can be conduction & convection depending up on in the presence of bulk fluid motion.

→ Absence of bulk fluid motion = conduction
presence of bulk fluid motion = convection

→ Heat transfer by convection > conduction because higher fluid velocity, higher the rate of Heat transfer.

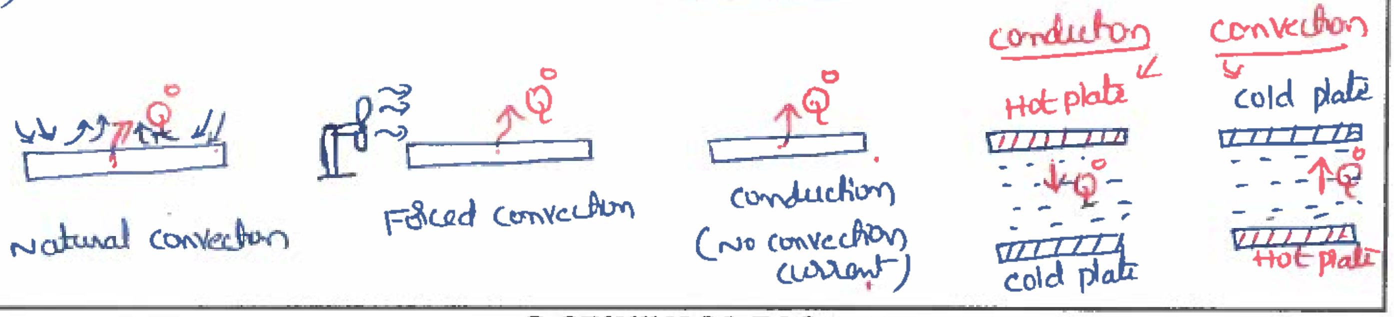
→ convection Heat transfer strongly depends on the fluid properties ie, Dynamic viscosity (μ), Thermal conductivity (K), density (ρ), specific heat (c) and fluid velocity (V) and also depends on geometry of body ie, surface roughness and type of fluid flow.

→ So that convection is most complex mechanism of HT, because so many parameters are involved.

→ According to newton's law of cooling

$$\boxed{\dot{Q}_{conv} = hA_s(T_s - T_\infty)}$$

→ The convection may either Natural convection & forced convection



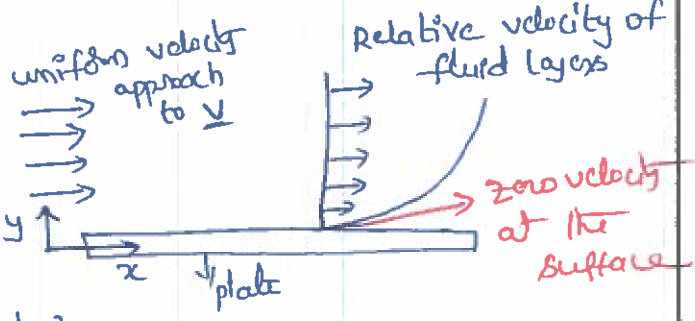
→ The convection co-efficient h depends on the several of the mentioned variables and thus it is difficult to determine

→ All the experimental observations indicates that fluid motion comes to a complete stop at the surface of the body and Assumes zero velocity

→ Fluid is directly contact with a solid, sticks to the surface due to viscosity effect and there is no slip ~~condition~~ is known as No-slip condition

→ Due to No-slip condition, the fluid layer at adjacent of surface is by pure conduction (zero velocity)

$$\boxed{\dot{q}_{cond} = -K_{fluid} \left.\frac{dT}{dy}\right|_{y=0}}$$ → W/m²



$$\therefore \left.\frac{dT}{dy}\right|_{y=0}$$

→ ∴ Heat transfer away from the surface is convection

$$\boxed{\dot{q}_{conv} = h(T_s - T_\infty)} \quad \text{— W/m}^2$$

→ ∴ Energy balance

$$\dot{q}_{cond} = \dot{q}_{conv}$$

$$-K_f \left.\frac{dT}{dy}\right|_{y=0} = h(T_s - T_\infty)$$

$$\boxed{h_x = \frac{-K_{fluid} \, (dT/dy)|_{y=0}}{T_s - T_\infty}} \quad \text{— W/m}^2\text{K}$$

$\therefore h_x$ = Local heat transfer co-efficient at a certain position x in flow direction and given temperature distribution

→ Local heat transfer co-efficient may vary ~~long~~ along the Length of flow as a result of changes in the velocity and other parameters in the flow direction.

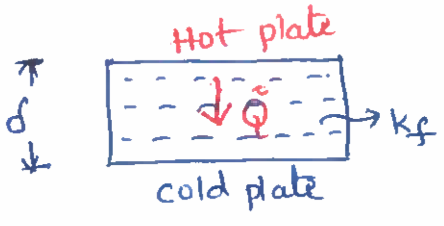
$$\boxed{h = \frac{1}{L}\int_0^L h_x \cdot dx}$$

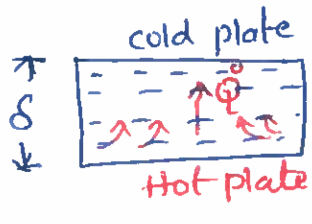
$\therefore$ from $x=0$ to $x=L$ and width W as

$$\boxed{Q = h\cdot(WL)(T_s - T_\infty)}$$

Nusselt Number :- [Nu]

It is the Ratio of heat convection to heat conduction (or) Ratio of conduction Resistance to convection Resistance





Conduction

$$\dot{Q}_{cond} = -KA\frac{dT}{dx}$$

$$= K_f A \frac{\Delta T}{\delta}$$

Convection

$$\dot{Q}_{conv} = hA\Delta T$$

$$\therefore \frac{Q_{conv}}{Q_{cond}} = \frac{hA\Delta t}{K_f A\frac{\Delta t}{\delta}} = \frac{h\delta}{K_f}$$

$$\therefore \boxed{Nu = \frac{Q_{conv}}{Q_{cond}} = \frac{h\delta}{K_f}}$$

$$\frac{R_{cond}}{R_{conv}} = \frac{L/KA}{1/hA}$$

$$\boxed{Nu = \frac{hL}{K}}$$

→ if Nu ↑ then h ↑ ⇒ ↑h ↑ $\dot{Q}_{conv}$

→ if Nu = 1 indicates Heat transfer by pure conduction only.

→ convection heat transfer is closely tied with fluid mechanics there are many fluid flows problems

1) Viscous and Inviscous Region flow
↳ At the surface mode viscous

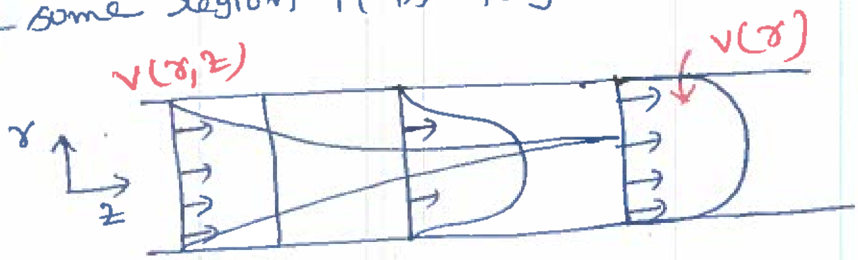
2) Internal and External flow.

3) compressible and incompressible flow.
↓ $\rho \neq$ constant
↓ ρ = const

4) Laminar and Turbulent flow.
↓ Smooth layers of fluid
↓ High disordered fluid motion.

5) steady and unsteady flow.
↓ No change at any point with time
↓ Changes with time

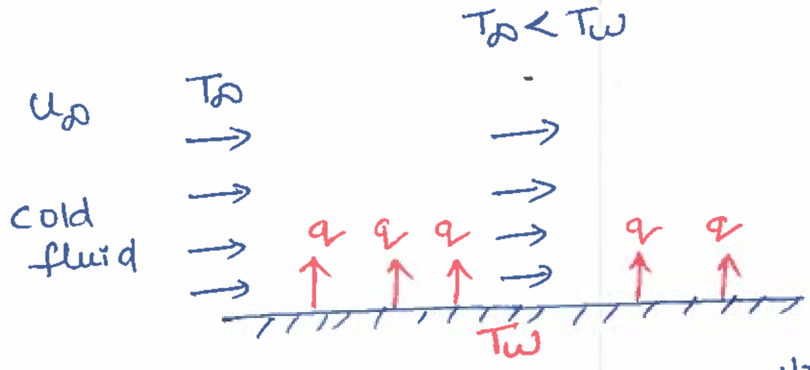
6) One, Two, Three Dimension flow.
↳ at enterance of pipe velocity is two dimensional
after some region it is fully developed and it is 1-D



7) Natural and Forced Flow.
↓ Due to Buoyancy effect
→ some external fluid motion involved

Convection Boundary Layers :-

→ To understand the convection heat transfer, let us consider a flat surface at a temparature T_w and a stream of cold fluid ~~flows T_∞~~ flowing though this surface with the temp of T_∞, which is Less than T_w. as shown in fig



→ Heat is flowing from the surface ⊥r to the flat plate [Heat flux = Heat flow / unit area]. if we increase the ~~co~~ velocity (U_∞) of cold fluid the plate ~~is~~ ~~cooler~~ ~~co~~ wills cools faster and if velocity decreases, the plate wills cools very slower.

→ This means flow of Heat flux is depends up on the flow of the cold fluid. if we ~~der~~ free stream velocity (U_∞) of the Rate of HT decreas the flow of fluid it decreases the Rate of HT (or) rate of heat flux.

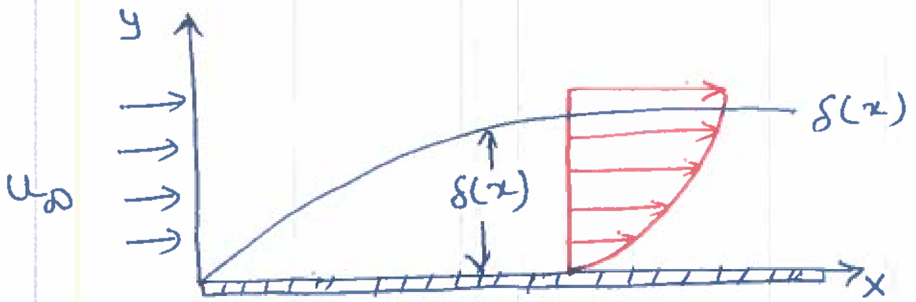
→ Therefore the flud velocity some effects on Heat flux on this plate

→ for Better understanding convection mechanism, we shard Learn a deeper mechanism of velocity boundary Layer and Thermal boundary Layer.

1) velocity Boundary Layer (δ) :-

→ Consider the parallel flow of a fluid over a flat plate as shown in fig.

→ It is characterized by presence of velocity gradient and shear stress



→ when the fluid flows over a plate, the velocity of the particles in the first fluid layer adjacent to the plate becomes zero, because of the no slip condition.

→ This motionless layers slows down the particles of the neighboring fluid layer as a result of friction (shear stress) effect b/w the particles of these two adjoing fluid layers at different velocities.

→ This fluid layers then slows down the molecules of the next layer, and so on. until a distance $y = \delta$ from the surface reaches, where these effects becomes negligible and the fluid velocity "u" reaches the free stream velocity u_∞.

→ As a result of frictional effects b/w the fluid layers, the local fluid velocity u will vary from $x=0, y=0$ to $y=\delta$

→ This Region in which the velocity gradient is known as Hydrodynamic Boundary Layer (or) simply Boundary Layer.

→ ~~The Relata~~ The thickness of boundary layer δ is generally defined as a distance from the surface at which local velocity $u = 0.99$ of of free stream velocity [This means 99% of fluid approaches as free stream]

→ The Retardation of fluid motion in the boundary Layer is due to the shear (viscous) stress acting in opposite direction

→ with increasing the distance y from the surface, shear stress decreases, the local velocity u increases until approaches u_∞ and boundary thickness grows (δ increase with x)

→ Experimental studies indicates that the shear stress for most fluids is proportional to the velocity gradient and the shear stress at the wall surface is expressed as

$$\boxed{\tau = \mu \cdot \frac{du}{dy}\Big|_{y=0}} \quad - N/m^2$$

→ The surface shear stress τ_s in terms of friction co-efficient C_f is expressed as

$$\boxed{\tau_s = \frac{C_{fx}}{2} \cdot \rho u_\infty^2}$$

where

τ_s = shear stress $- N/m^2$

C_{fx} = skin friction co-efficient

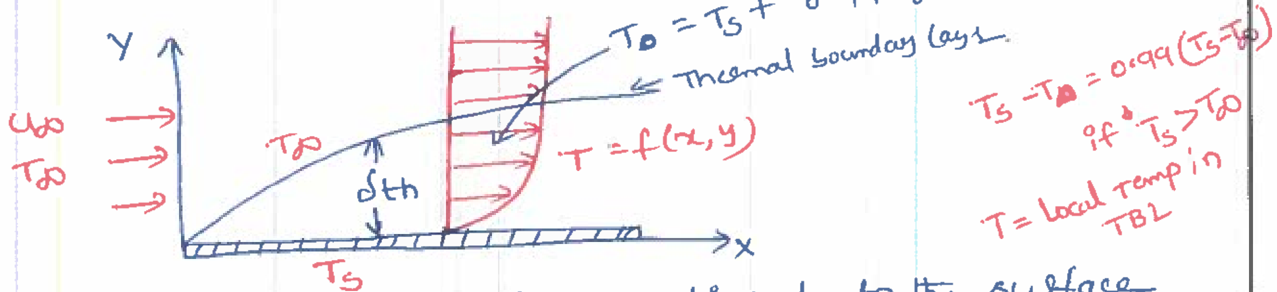
ρ = Density $- kg/m^3$

u_∞ = free stream velocity $- m/s$

$\frac{du}{dy}$ = velocity gradient

2) Thermal Boundary Layer (δ_{th}) :-

→ consider the flow of a fluid at ~~~~ a uniform temperature T_∞ over the an isothermal flat plate at Temperature T_s



→ The fluid particles in the layer adjacent to the surface reach thermal equilibrium with the plate and assume the surface temperature T_s.

→ These fluid particles then exchange energy with the particles in the adjoing fluid layer, and so on. and as this Result a temperature profile develops in the flow field that ranges from ~~at the~~ T_s at the surface to T_∞ sufficiently far from the surface. ~~This surface~~

→ The flow region over the surface in which the temperature variation in the direction, normal to the surface is observed is called thermal Boundary Layer.

→ with increasing the distance from leading edge the effect of heat transfer penetrates further into the free stream and the thermal boundary Layer grows as shown in fig. above.

→ The shape of the temperature profile in the thermal boundary Layer indicates the convection heat transfer b/w a solid surface and fluid flowing over it.

→ In flow over a heated (or cooled) surface, both velocity and thermal boundary layers develop simultaneously.

→ Noting the the fluid velocity has a strong influence on the temperature profile, the development of the velocity boundary Layer relative to the thermal boundary layer will have strong effect on the convection heat transfer.

Significance of Boundary Layers :-

→ The velocity boundary Layer ($\delta(x)$) is characterized by presence of velocity gradient and fluid friction. The Thermal boundary Layer ($\delta_{th}(x)$) is characterized by temp gradient and heat transfer.

→ For flow over ~~head~~ heated (or) cooled surface, both velocity and thermal boundary Layers are developed simultaneously.

→ The thermal boundary Layer thickness may be Lesser than the velocity boundary layer, may be greater than the velocity boundary layer, may be equals to velocity boundary Layer i.e,

$$\delta_{th} < \delta$$

$$\delta_{th} > \delta$$

$$\delta_{th} = \delta$$

→ If the effects of fluid viscosity is stronger than the thermal effects, then the velocity boundary Layer will thicker than the thermal boundary Layer and vice versa.

→ The Relative thicknesses of the velocity and thermal boundary Layer is best described by the dimensionaless parameter prandtl number.

$$\therefore \text{Prandtl Number } (P_r) = \frac{\delta}{\delta_{th}}$$

δ = momentum diffusivity $- \nu = \mu/\rho$

δ_{th} = Heat diffusivity $= \alpha = K/\rho C_p$

$$\therefore \boxed{P_r = \frac{\delta}{\delta_{th}} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{K/\rho C_p}}$$

→ Therefore both μ and ρ, ~~both~~ combinedly decides the velocity boundary layer thickness and both K and ρC_p combinedly decides the thermal boundary layer thickness

μ = Dynamic viscosity (or) simply viscosity

ρ = density

K = thermal conductivity

ρC_p = Heat capacity

Prandtl Number :- [Pr]

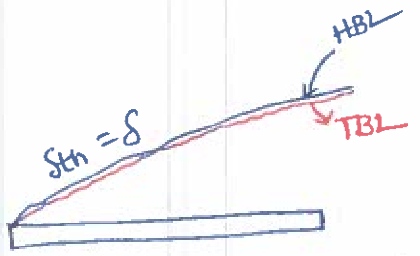
→ It is the Ratio of momentum diffusivity to thermal diffusivity

$$\boxed{P_r = \frac{\nu}{\alpha} = \frac{\mu/\rho}{K/\rho C_p} = \frac{\mu C_p}{K}}$$

→ It is purely depends on fluid properties

→ It gives Inter relation b/w Hydrodynamic boundary Layer (HBL) and Thermal boundary Layer (TBL)

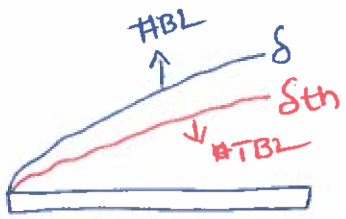
case 1 :- $P_r = 1$



→ HBL Merges in TBL

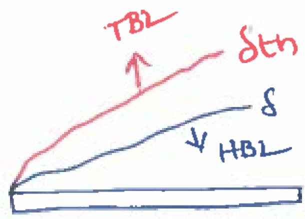
→ generally for gases (air) $P_r = 1$

Case 2 :- $P_r > 1$ [Lubricate oil]



$$P_r = \frac{\nu}{\alpha} > 1$$

Case 3 :- $P_r < 1$ [Mercury]



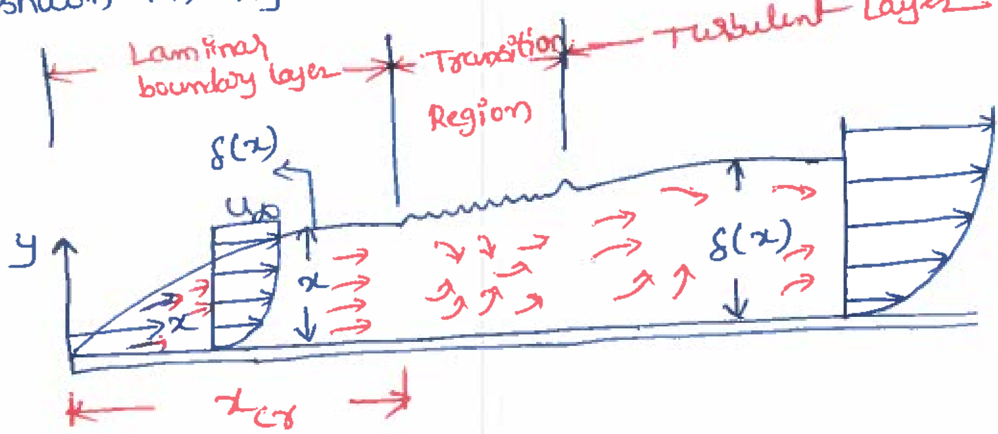
$$P_r = \frac{\nu}{\alpha} < 1$$

Laminar and Turbulent flow :-

→ The Analysis of convection problems requires the knowledge of the type of boundary Layer developed, whether it is Laminar (or) Turbulent

→ The Type of boundary strongly influences the skin friction (C_{fx}) and heat transfer co-efficient (h)

→ The developed boundary Layer may consists of Laminar boundary, transition region and turbulent boundary Layer as shown in fig below



→ The velocity boundary Layer $\delta(x)$ is characterized by the presence of velocity gradient and shear stress

→ The Thermal boundary Layer $\delta_{th}(x)$ is characterized by temparature gradient and heat transfer

1) Laminar Boundary Layer :-

→ velocity boundary Layer starts the Leading edge of the plate as a Laminar boundary Layer, in which fluid motion is highly ordered, it is a stream line

→ velocity component $u \rightarrow x$-direction
$v \rightarrow y$-direction

→ velocity profile near to parabolic.

2) Turbulent Boundary Layer :-

→ In this fluid motion has very large disturbances and is by velocity fluctuation

→ velocity fluctuations increases the momentum and Heat transfer
∴ if $u\uparrow$ momentum ↑ & HT ↑

→ Due to fluid mixing, turbulent boundary Layer thickness $(\delta(x))$ is large (↑), velocity profile is ~~faster~~ flatter

→ In the flow from the Leading edge small disturbance in flow begains to starts and fluctions begains = transition Region (b/w Laminar & turbulent).

→ The transition to turbulence is attained by significant increase in boundary Layer thickness, wall shear stress, and heat transfer co-efficient
∴ From transition to Turbulence ↑ δ & ↑ HT

→ The characteristics of fluid flow is governed by dimensionless quantity called Reynolds number.

Reynolds Number (Re) :-

→ The transition from laminar to turbulent flow depends on the surface geometry, surface roughness, flow velocity, surface temp and type of fluid.

→ ∴ Reynolds is " Ratio of inertia force to viscous force of the fluid "

$$\boxed{Re = \frac{\text{Inertia Force}}{\text{Viscous force}} = \frac{u_\infty x}{\nu} = \frac{\rho V L_c}{\mu}}$$

$u_\infty = V$ = free stream velocity – m/s

ν = kinematic viscosity – m^2/s [Like (α) – m^2/s]

$x = L_c$ = characteristic Length = distance from the Leading edge for flow over a flat plate – m

→ The Reynolds number at which flow becomes turbulent is called Critical Reynold's Number (Re_{cr})

$$\boxed{Re_{cr} \cong 5 \times 10^5}$$

↳ For flow over Flat plate (or) External fluid flow over flat surfaces

Conventional Questions :-

1) water flows at $20°C$ at $8\ kg/s$ through the diffuser having 3cm diameter at the enterance and 7cm at its exit. calculate the fluid velocity and Reynolds number at the Inlet & exit of diffuser.

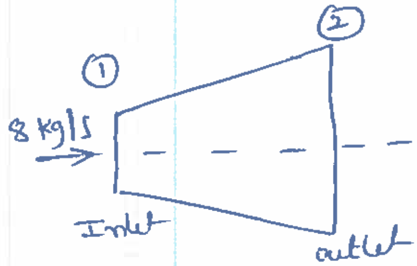
Data

$D_1 = 3cm = 0.03m$

$D_2 = 7cm = 0.07m$

$T = 20°C$

$\dot{m} = 8\ kg/s$



Find

1) $V_1 = ?,\ V_2 = ?$

2) $Re_1 = ?,\ Re_2 = ?$

Sol

from Discharge

$$Q = A \times V$$

$$V = \frac{Q}{A} \quad \frac{m^3/s}{m^2}$$

$$V = \frac{\dot{m}}{\rho} / A$$

$$\rho = \frac{m}{V}$$

$$V = \frac{m}{\rho} \quad - m^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} \quad - m^3/s$$

$$\therefore V_1 = \frac{\dot{m}_1}{\rho_1 A_1}$$

from data book at $20°C$ of water
$\rho = 1000\ kg/m^3$

$$V_1 = \frac{8}{1000 \times (\frac{\pi}{4} \times 0.03^2)} \Rightarrow V_1 = 11.32\ m/s$$

$$V_2 = \frac{\dot{m}_2}{\rho_2 A_2} = \frac{8}{1000 \times \frac{\pi}{4}(0.07)^2}$$

$$V_2 = 2.08\ m/s$$

2)

Reynold's Number (Re)

$$Re = \frac{\rho V D}{\mu} \text{ for pipe flow}$$

$$= \frac{VD}{\nu}$$

$\nu = \frac{\mu}{\rho}$

$\mu = \nu \times \rho$

from data book at 20°C of water

$$\nu = 1.006 \times 10^{-6}\ m^2/s$$

$$Re_1 = \frac{V_1 d_1}{\nu_1}$$

$$= \frac{11.32 \times 0.03}{1.006 \times 10^{-6}}$$

$$Re_1 = 3.37 \times 10^5$$

$Re_{cr} = 2300$ for Internal flow

$$Re_2 = \frac{V_2 d_2}{\nu_2}$$

$$= \frac{2.08 \times 0.07}{1.006 \times 10^{-6}}$$

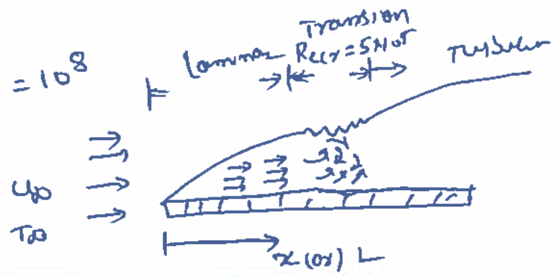
$$Re_2 = 1.44 \times 10^5$$

2) A Fan provides air speed upto 50 m/s is used in low speed wind tunnel with atmospheric air at 27°C. If this wind tunnel is used to study the boundary layer behaviour over a flat plate upto $Re = 10^8$, what should be the Minimum plate length? At what distance from the leading edge would transition occur, if critical Reynolds number is $Re_{cr} = 5 \times 10^5$?

Data $V = 50\ m/s$, $T_\infty = 27°C$, $Re = 10^8$

Find 1) $x = ?$ at $Re = 10^8$

2) $x_{cr} = ?$ if $Re_{cr} = 5 \times 10^5$



Sol property values are taken from data book at 27°C of air

$\rho = 1.16\ kg/m^3$, $\mu = 184.6 \times 10^{-7}\ kg/ms$

1) $Re_x = \dfrac{\rho V x}{\mu}$

$x = \dfrac{Re_x \times \mu}{\rho V}$

$= \dfrac{10^8 \times 184.6 \times 10^{-7}}{1.16 \times 50}$

$\therefore x = \underline{\underline{31.82\ m}}$

2) $Re_{cr} = \dfrac{\rho V x_{cr}}{\mu}$

$x_{cr} = \dfrac{Re_{cr}\, \mu}{\rho V}$

$= \dfrac{5 \times 10^5 \times 184.6 \times 10^{-7}}{1.16 \times 50}$

$\therefore x_{cr} = \underline{\underline{0.159\ m}}$

$\therefore$ transition from laminar to turbulent will occur at $\underline{\underline{x = 0.159\ m}}$

Physical significance of the Dimensionless parameters :-

1) Nusselt Number [Nu]

→ It is defined as the ratio of convection heat flux ($\dot{q}_{conv}$) to conduction heat flux ($\dot{q}_{cond}$) in the fluid boundary layer

$$\boxed{Nu = \frac{\text{Convection Heat flux}}{\text{Conduction Heat flux}} = \frac{h\,\Delta T}{k_f\,\Delta T/L_c} = \frac{hL_c}{k_f}}$$

where

ΔT = temp difference $(T_s - T_\infty)$ b/w wall surface and fluid – K

h = heat transfer co-efficient – W/m^2K

L_c = Thickness of fluid Layer (characteristic length) – m

k_f = Thermal conductivity of fluid – W/mK.

→ if $Nu = 1$ ⇒ There is no convection, the HT is pure conduction

→ Larger value of Nu indicates larger the ~~conduction~~ convection in the fluid

$$Nu \uparrow \Rightarrow h \uparrow$$

2) Prandtl Number (P_r) :-

→ It is defined as, the ratio of momentum diffusivity (ν) to Thermal diffusivity (α)

$$\boxed{P_r = \frac{\text{momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{\mu c_p}{k_f}}$$

→ It is the function of temperature

→ ⊖

→ It measures of Relative effectiveness of momentum and Energy transfer in the velocity and thermal boundary layers

→ For gases $P_r \cong 1$ ⇒ both ν & α is takes place at same rate

→ For Liquid metals $P_r << 1$ ⇒ Heat diffusion (α) is very fast (↑)

→ For oils $P_r >> 1$ ⇒ Heat diffusion (α) is very slow(↓)

→ ∴ The Thermal boundary Layer thickness $\delta_{th}(x)$ is high for Liquid metals, very thin for oils

$$\boxed{P_r^n = \frac{\delta_{th}(x)}{\delta(x)}}$$

where n = exponent

3) Reynold's Number (Re) :-

→ Ratio of inertia force to viscous force & it is used in forced convection

$$\boxed{Re = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{\rho U_\infty L_c}{\mu} = \frac{U_\infty L_c}{\nu}}$$

L_c = Characteristic Length of flow geometry – m

= x, distance from the Leading Edge in the flow direction for a flat plate

= D, ~~Distance~~ Diameter for flow through (or) across a cylinder and sphere

u_∞ = free stream velocity - m/s

μ = Dynamic viscosity of fluid - Ns/m^2 (or) $kg/m\text{-}s$

$\nu = \mu/\rho$ = Kinematic viscosity - m^2/s

→ It tells about type of flow whether Laminar (or) Turbulent flow.

4) Critical Reynold's Number (Re_{cr}) :-

→ It is the value of Reynold's number, where boundary Layer ~~thickness~~ changes from Laminar to Turbulent nature

→ for flow over Flat plate

$$\boxed{Re_{cr} \geq 5 \times 10^5}$$

→ for flow through tubes

$$\boxed{Re_{Dcr} = \frac{u_\infty D}{\nu} \geq 2300}$$

→ Re_{cr} depends upon
1) surface roughness
2) Level of turbulence
3) variation of pressure along the flow.

5) Grashof Number (Gr) :-

→ It is defined as the ratio of buoyancy force to viscous forces acting in the fluid Layer

→ It is a major role in natural convection

$$\boxed{Gr = \frac{g \beta L_c^3 \Delta T}{\nu^2}}$$

g = acceleration due to gravity $- m/s^2$

β = co-efficient of volumetric expansion

$= \frac{1}{(T_f + 273)} - K^{-1}$

ΔT = Temp difference b/w surface and fluid $- °C$ (or) K

T_f = Mean film temperature $= \frac{T_s + T_\infty}{2} - °C$

ν = Kinematic viscosity of fluid $- m^2/s$

L_c = significant length of the body $- m$

= height, L for vertical plate and cylinder

= Distance, D for horizontal cylinder and sphere

$= \frac{\text{surface Area}}{\text{perimeter}} = \frac{A_s}{P}$ → for other geometry

→ For free convection, the transition from laminar to turbulent occurs, when $Gr_{cr} \simeq 10^9$

6) Stanton number (St_x) :-

→ It is the ratio of heat transfer at the surface to that transported by fluid it's Thermal capacity

$$St_x = \frac{\text{Heat flux to the fluid}}{\text{Heat transfer capacity of fluid}}$$

$$= \frac{h \Delta T}{\rho c_p u_\infty \Delta T} = \frac{h}{\rho c_p u_\infty}$$

$$\frac{\frac{hL}{K}}{\frac{\rho V D}{\mu} \times \frac{\mu c_p}{K}}$$

$$\frac{hK}{K} \times \frac{\mu \times K}{\rho V \mu \times \mu c_p}$$

$$\frac{h}{\rho c_p u_\infty}$$

$$St_x = \frac{Nu_x}{Re_x \cdot Pr}$$

ie, $= \dfrac{\frac{hL_c}{K}}{\frac{\rho V L_c}{\mu} \times \frac{\mu C_p}{K}}$

$= \dfrac{h\cancel{L_c}}{\cancel{K}} \times \dfrac{\cancel{\mu}\,\cancel{K}}{\rho V \cancel{L_c}\,\cancel{\mu} C_p}$

$= \dfrac{h}{\rho C_p V}$

7) peclet number (Pe) :-

→ It is the "Ratio of heat transfer by convection to heat transfer by conduction

$$\boxed{Pe = \frac{m c_p \Delta T}{K A \Delta T / L}}$$

$$= \frac{(\rho \cancel{A} u) C_p \cancel{\Delta T}}{K \cancel{A}\, \cancel{\Delta T} / L}$$

$$\boxed{Pe = \frac{\rho u C_p L}{K_f}}$$

$Q_{conv} = m c_p \Delta T$

This mean whatever conduction takes place by body is converted to ~~surround~~ surrounding, this is stored in surrounding $Q_{st} = m c_p dT$

→ Peclet can be Reset as

$$Pe = \frac{\rho u C_p L}{K_f} \times \frac{\mu}{\mu}$$

$$= \frac{\rho V L}{\mu} \times \frac{\mu C_p}{K_f}$$

$$\boxed{Pe = Re \times Pr}$$

∴ Peclet number is the function of Reynold's number and Prandtl's number

8) Graetz Number (G_z) :-

→ It is the Ratio of heat capacity of fluid flowing through the pipe per unit length to conductivity of pipe material

(or)

→ ~~Ratio of fluid stream thermal~~ capacity ~~to convective heat~~

$$\boxed{G_z = \frac{mc_p/L}{K} = \frac{mc_p}{KL}} =$$

$$\Rightarrow \frac{mc_p}{LK}$$

→ G_z can be Reset as

$$G_z = \frac{mc_p}{KL}$$

$$= \frac{(\rho AV)\,c_p}{KL}$$

$$= \frac{\rho\left(\frac{\pi}{4}d^2 \times v\right)c_p}{KL} \times \frac{\mu}{\mu}$$

$$= \frac{\pi}{4}\,\frac{\rho V d}{\mu} \times \frac{\mu c_p}{K} \times \frac{d}{L}$$

$$\boxed{G_z = \frac{\pi}{4}\left(Re \cdot Pr\right)\frac{d}{L}}$$

where d and L are diameter and length of pipe.

→ G_z can Reset as another form

$$G_z = \frac{mc_p}{KL} = \frac{(\rho AV)\,c_p}{KL} = \frac{AV}{\alpha \cdot L}$$

$$= \frac{\pi}{4}d^2\,\frac{V}{\alpha L} = \frac{Vd}{\alpha}\left(\frac{\pi d}{4L}\right)$$

$$\boxed{G_z = Pe\,\frac{\pi d}{4L}}$$

1) calculate The approximate Reynolds number and state if the flow is Laminar or turbulent.

i) A 10m long yatch sailing at 13km/hr in sea water $\rho = 1000$ kg/m^3 and $\mu = 1.3 \times 10^{-3}$ kg/ms

ii) A compressor disc of radius 0.3m rotating at 15000 rpm in a 5bar and 400°C and $\mu = \dfrac{1.46 \times 10^{-6} T^{3/2}}{(110+T)}$ kg/ms.

iii) 0.05kg/s of CO_2 gas at 400K flowing in a 20mm dia pipe and $\mu = \dfrac{1.56 \times 10^{-6} T^{3/2}}{(233+T)}$ kg/ms.

T is in Kelvin

~~Data~~ Sol

i) $L = 10m$, $u_\infty = 13 km/hr$

$= \dfrac{13 \times 1000}{3600} m/s = 3.61 m/s$

$\mu = 1.3 \times 10^{-3}$ kg/ms, $\rho = 1000$ kg/m^3

$$Re = \frac{\rho u_\infty L}{\mu} = \frac{1000 \times 3.61 \times 10}{1.3 \times 10^{-3}}$$

$$Re = 2.78 \times 10^5$$

$\therefore Re > Re_{cr} (5 \times 10^5)$ $\therefore$ flow is Laminar

ii) $R = 0.3m$ $\quad N = 15000$ rpm $\quad T = 400°C$

$D = 0.6m$ $\quad P = 5 bar$

$= 5 \times 10^5 Pa$

$= 500 kPa$

$\mu = \dfrac{1.46 \times 10^{-6} T^{3/2}}{(110+T)}$ kg/m

$$Re = \frac{\rho u_\infty L}{\mu}.$$

from ideal gas Equation

$$PV = mRT$$

$$P = \frac{m}{V} RT \Rightarrow P = \rho RT \Rightarrow \rho = \frac{P}{RT}$$

$$\rho = \frac{500}{0.287 \times (400+273)}$$

$$= 2.588 \ kg/m^3.$$

$$\mu = \frac{1.46 \times 10^{-6} (400+273)^{3/2}}{(110 + (400+273))}$$

$$\mu = 3.3 \times 10^{-5} \ kg/ms$$

$$u_\infty = \frac{\pi D N}{60} = \frac{\pi \times 0.6 \times 15000}{60} =$$

$$\therefore Re = \frac{2.588 \times \quad \times 0.6}{3.3 \times 10^{-5}} =$$

$\therefore Re > Re_{cr}$ $\therefore$ flow is Turbulent

iii) Internal flow

$\dot{m} = 0.05$ kg/s, $T = 400k$, $D = 20mm = 20 \times 10^{-3} m$

$$\therefore Re = \frac{\rho u_\infty D_h}{\mu} = \frac{\rho u}{\mu} \frac{4A_c}{P}$$

$D_h = \frac{4A_c}{P}$

$$= \frac{4 \rho A_c u}{\mu P} = \frac{4 \dot{m}}{\mu \pi D}.$$

$\dot{m} = \rho A V$

$$\therefore \mu = \frac{1.56 \times 10^{-6} (400)^{3/2}}{(233+400)}$$

$$= 1.97 \times 10^{-5} \ kg/ms$$

$$\therefore Re = \frac{4 \times 0.05}{1.97 \times 10^{-5} \times \pi \times 20 \times 10^{-3}}$$

$$Re = 161578.62$$

$\therefore Re > Re_{cr}$ (2300 for Internal flow)

$\therefore$ Flow is Turbulent